Exercise 77

(a) If we shift a curve to the left, what happens to its reflection about the line $y = x$? In view of this geometric principle, find an expression for the inverse of $g(x) = f(x + c)$, where f is a one-to-one function.

(b) Find an expression for the inverse of $h(x) = f(cx)$, where $c \neq 0$.

Solution

Because f is a one-to-one function, its inverse f^{-1} exists.

Part (a)

Shifting a curve to the left makes its reflection across the $y = x$ line move down. Solve the given equation for x in terms of $g(x)$.

$$g(x) = f(x + c)$$

Apply the inverse of f to both sides.

$$f^{-1}[g(x)] = f^{-1}[f(x + c)]$$

f^{-1} undoes f on the right side.

$$f^{-1}[g(x)] = x + c$$

The inverse function is then

$$x = f^{-1}[g(x)] - c.$$

One could make it its own function by calling x g^{-1} and replacing $g(x)$ with x.

$$g^{-1} = f^{-1}(x) - c$$

Part (b)

Solve the given function for x.

$$h(x) = f(cx)$$

Apply the inverse of f to both sides.

$$f^{-1}[h(x)] = f^{-1}[f(cx)]$$

f^{-1} undoes f on the right side.

$$f^{-1}[h(x)] = cx$$

The inverse function is then

$$x = \frac{1}{c} f^{-1}[h(x)].$$

One could make it its own function by calling x h^{-1} and replacing $h(x)$ with x.

$$h^{-1} = \frac{1}{c} f^{-1}(x)$$